Aeroassisted Transfer Between Elliptical Orbits Using Lift Control

David Mishne*
Rafael, Haifa, Israel
and
Nahum Melamed† and Josef Shinar‡
Technion—Israel Institute of Technology, Haifa, Israel

A transfer between two elliptical orbits, using an atmospheric passage, is considered. In the atmosphere, the lift of the spacecraft is controlled to change the line of apsides of the orbit, without changing other orbital parameters. The maneuver includes three propulsive impulses to bring the spacecraft into and out of the atmosphere and to compensate for the velocity loss during the atmospheric passage. The total velocity change during this maneuver is compared to the velocity change of an optimal two-impulse, pure-propulsive maneuver and of a drag-only aeroassisted maneuver. It is shown that below a certain apsidal rotation angle, the lift-controlled maneuver is more fuel-economic than the drag-only maneuver. The lift-controlled maneuver is also more fuel economic than the pure-propulsive maneuver, provided that the lift-to-drag ratio of the spacecraft is greater than two. The limits to the rotation angle that can be achieved in a single pass are discussed. Numerical examples are presented.

Introduction

THE aeroassisted maneuver (a combined propulsive and aerodynamic maneuver) has been a continuous research subject since its first presentation in 1962.[1] A recent survey paper[2] discusses the state of the art of this subject. It has been shown that under certain conditions, an aeroassisted maneuver requires less fuel than a pure-propulsive maneuver. Mostly, two types of maneuvers were discussed: the transfer from a high orbit to a low orbit[3] and the plane change maneuver.[4]

Another maneuver that is of interest is the transfer between two coplanar elliptical orbits. When the major axes and the eccentricities of the orbits are the same, the maneuver is simply a planar rotation of the line of apsides. This maneuver may be required, for example, to compensate for the change in the argument of perigee due to the Earth's oblateness. Naturally, it is required to minimize the fuel consumption of this maneuver.

The transfer between two coplanar orbits, using only thrust impulses, is discussed in Ref. 5. Vinh and Johannesen[6] presented an aeroassisted transfer that uses drag only. Two methods for drag-only aeroassisted transfers were presented: 1) using the atmosphere to circularize the orbit and then raising the orbit to its original semimajor axis at the new rotation angle of the line of apsides, and 2) transferring the spacecraft to a parabolic orbit, rotating the line of apsides, and then using the atmosphere to brake the parabolic orbit down to the original orbit.

It was shown in Ref. 6 that, for large rotation angles, the aeroassisted transfers use less fuel than the optimal two-impulse maneuver.

A spacecraft that is capable of lift generation can perform another aeroassisted maneuver in which the lift is controlled to change the line of apsides in a single atmospheric pass. In this paper we discuss this maneuver. We show under what conditions this maneuver is more fuel-economic than either the pure-propulsive maneuver or the aeroassisted, drag-only maneuver.

Aeroassisted Transfer Using Lift Control

Consider an initial elliptical orbit with given semimajor axis a and eccentricity e. It is required to rotate the line of apsides by an angle of $\Delta\omega$ without changing either a or e.

The lift-controlled, aeroassisted transfer is performed as follows (see Fig. 1):

1) A tangential impulse ΔV_1 is applied at the apogee to lower the perigee from r_p to r_p'. The r_p' is inside the atmosphere, which is characterized by the radius R_a below which the aerodynamic forces are significant. The ΔV_1 is chosen so that a certain entry flight path angle γ_i is achieved at R_a (usually a few degrees). It should be noted that r_p' is not the actual perigee, since the trajectory portion inside the atmosphere is not Keplerian.

2) A lift control law is applied to change the angle of attack of the spacecraft. The spacecraft moves inside the atmosphere toward an exit point f.

3) A tangential impulse ΔV_2 is applied at the exit point. This impulse compensates for the velocity loss during the atmospheric passage and brings the apogee of the orbit to the initial one r_a.

4) A final impulse is applied at the apogee to raise the perigee to the initial one r_p.

The new orbit has the same semimajor axis and same eccentricity as the initial one, but has a different line of apsides. The rotation angle is $\Delta\omega$.

The aeroassisted, lift-controlled maneuver then includes three thrust impulses and an atmospheric passage. The total fuel consumption is related to the sum of the impulses:

$$\Delta V_{\text{total}} = \Delta V_1 + \Delta V_2 + \Delta V_3 \tag{1}$$

The maneuver is controlled by 1) ΔV_1, which controls the entry conditions, and 2) the angle of attack law, which is the control variable during the atmospheric passage.

In this paper we discuss this aeroassisted, lift-controlled maneuver. The resulting total ΔV is compared to the alterna-

Presented as Paper 88-4346 at the AIAA Atmospheric Flight Mechanics Conference, Minneapolis, MN, Aug. 15–17, 1989; received Nov. 1, 1988; revision received Aug. 3, 1989.

*Senior Research Engineer. Senior Member AIAA.
†Graduate Student, Faculty of Aerospace Engineering.
‡Professor, Faculty of Aerospace Engineering. Associate Fellow AIAA.

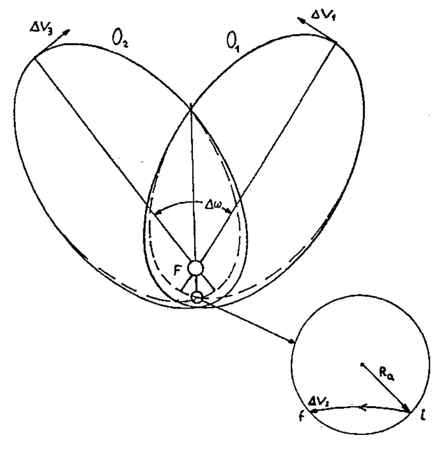

Fig. 1 Lift controlled aeroassisted maneuver.

tives (pure-propulsive maneuver and an aeroassisted, drag-only maneuver). The conditions and limits to the single pass ω change are discussed.

Approximate Solution

We first discuss an approximate solution to the problem by using simple expressions for the rate of change of the orbital parameters due to the effect of external forces.

The aerodynamic force per unit mass is decomposed to a tangential force f_t (positive forward) and a normal force f_n (positive downward). The rate of change of ω is[7]

$$e_i\dot{\omega} = (2f_t/V)\sin\theta + f_n(2e_i + r\cos\theta/a_i)/V$$

where V is the velocity, r the local radius, and θ the true anomaly. The a_i and e_i are the semimajor axis and the eccentricity, respectively, of the transfer ellipse with the initial apogee r_a and the reduced perigee r_p'.

The r is given by the ellipse equation

$$r = a_i(1 - e_i^2)/(1 + e_i\cos\theta)$$

The entry angle into the atmosphere is small. Hence it is reasonable to assume that in the atmospheric portion of the trajectory, θ is small. Then

$$r \approx r_p' = a_i(1 - e_i) \tag{2a}$$

$$e_i\dot{\omega} = f_n(1 + e_i)/V \tag{2b}$$

The normal force, which is equivalent to negative lift force, is responsible for the change of ω. A negative lift increases ω, i.e., the line of apsides rotates in the same direction as the spacecraft, and vice versa. The drag force acts to reduce the velocity. In the atmospheric portion, we have approximately

$$\dot{V} = f_t \tag{3}$$

Dividing Eq. (3) by Eq. (2b), and remembering that $f_t/f_n = C_D/C_L$, we get

$$\dot{V}/\dot{\omega} = [e_iV/(1 + e_i)](C_D/C_L) \tag{4}$$

Assuming that the velocity loss is small compared to the velocity itself, we can replace V by the perigee velocity, which is given by

$$V_p = [\mu(1 + e_i)/a_i(1 - e_i)]^{1/2}$$

where μ is the gravitational constant.

Assuming constant C_D/C_L, Eq. (4) is integrated to obtain the velocity loss per unit rotation angle

$$\Delta V_2/\Delta\omega = e_i\sqrt{[\mu/a_i(1 - e_i^2)]}\,(C_D/C_L) \tag{5}$$

It is evident that $\Delta V_2/\Delta\omega$ is minimum when the ratio C_L/C_D is maximum.

To find the total velocity loss, we have to add to ΔV_2 the initial and final impulses.

The ΔV_1 is the velocity change that is required to change the perigee from r_p to r_p'. It is given by

$$\Delta V_1 = \sqrt{\frac{2\mu r_p}{r_a(r_a + r_p)}} - \sqrt{\frac{2\mu r_p'}{r_a(r_a + r_p')}} \tag{6}$$

We assume that the exit conditions, after the impulse ΔV_2 is applied, are the same as the entry conditions. Then we have $\Delta V_3 = \Delta V_1$. The total velocity change is the sum of the absolute values of the individual changes:

$$\Delta V_{\text{total}} = 2\sqrt{\frac{2\mu}{r_a}}\left[\sqrt{\frac{r_p}{r_a + r_p}} - \sqrt{\frac{r_p'}{r_a + r_p'}}\right] + e_i\sqrt{\frac{2\mu}{(r_a + r_p')(1 - e_i^2)}}\left|\frac{\Delta\omega}{C_L/C_D}\right|$$

Remembering that $r_a = a(1+e)$, $r_p = a(1-e)$, and $r_p' = a_i(1-e_i)$, we get

$$\left[\frac{\Delta V_{\text{total}}}{\sqrt{\mu/p}}\right]_{AL} = 2(1 - e)\left[1 - \sqrt{\frac{2}{n(1 + e) + 1 - e}}\right] + \left\{\frac{n(1 + e) - 1 + e}{\sqrt{2}\sqrt{[n(1 + e) + 1 - e]}}\right\}\left|\frac{\Delta\omega}{C_L/C_D}\right| \tag{7}$$

where $p = a(1 - e^2)$ is the parameter of the initial orbit, and $n \triangleq r_p/r_p'$ is the ratio of the initial perigee to the perigee of the entry orbit. Let us assume further that r_p' just grazes the atmosphere. Hence $n = r_p/R_a$.

Equation (7) yields the total velocity loss associated with the aeroassisted, lift-controlled maneuver (AL).

First, let us compare this result to the velocity loss associated with a two-impulse, pure-propulsive maneuver. The equations for the velocity loss for the pure-propulsive maneuver are given in Ref. 6. It is clear that above a certain C_L/C_D, the aeroassisted maneuver is advantageous. To get a preliminary guess of the minimum C_L/C_D required, we simplify the equations of the pure-propulsive maneuver. It can be shown (see Appendix) that for $e<0.4$ and $\Delta\omega<50$ deg, the velocity loss for the two-impulse, pure-propulsive maneuver (PM) is given by

$$\left[\frac{\Delta V_{\text{total}}}{\sqrt{\mu/p}}\right]_{PM} = e\,\Delta\omega/2 \tag{8}$$

Comparing this to Eq. (7) for $n = 1$, we observe that the aeroassisted maneuver is more fuel-economic if

$$(C_L/C_D)_{\max} > 2$$

This condition requires a highly aerodynamic efficiency (typical of a winged spacecraft). An exact comparison will be presented in the sequel, where we discuss the exact solution.

Next, let us compare the velocity loss in the lift-controlled maneuver [Eq. (7)] to the velocity loss associated with the drag-only maneuvers. Vinh and Johannesen[6] showed that

$$\left[\frac{\Delta V_{\text{total}}}{\sqrt{\mu/p}}\right]_{AE} = 2(1 - e)\left[1 - \sqrt{\frac{2}{n(1 + e) + 1 - e}}\right] - \sqrt{n(1 + e)} + n(1 + e)\sqrt{\frac{2}{n(1 + e) + 1 - e}} \tag{9}$$

$$\left[\frac{\Delta V_{\text{total}}}{\sqrt{\mu/p}}\right]_{PR} = \sqrt{2(1+e)} - 2e - (1-e)\sqrt{\frac{2}{n(1+e) + 1 - e}} \tag{10}$$

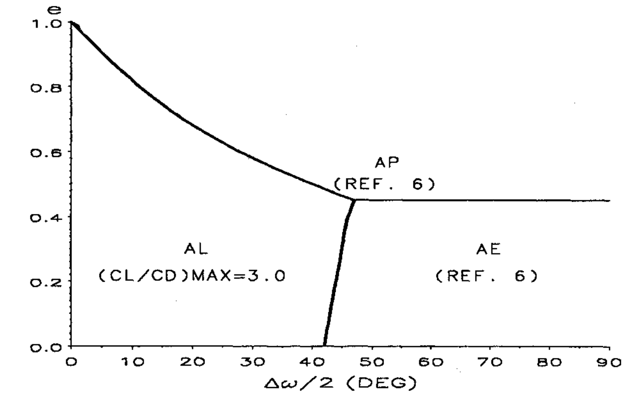

Fig. 2 Optimal regions—approximate solution, $n = 1$.

where $[\Delta V_{\text{total}}/\sqrt{(\mu/p)}]_{AE}$ is the total velocity loss for the drag-only elliptical re-entry transfer, and $[\Delta V_{\text{total}}/\sqrt{(\mu/p)}]_{PR}$ is the total velocity loss for the drag-only parabolic transfer.

For the two drag-only modes, it is clear from Eqs. (9) and (10) that, for a given value of n, there is a certain eccentricity below which the AE mode is preferable. Let us denote this eccentricity by e_c, which can be determined by equating Eqs. (9) and (10).

By comparing Eq. (7) to Eqs. (9) and (10), we see that the velocity loss for the lift-controlled maneuver depends upon the line of apsides rotation angle $\Delta\omega$, whereas the velocity loss for the drag-only maneuvers is independent of $\Delta\omega$. Therefore it is clear that below a certain value of $\Delta\omega$, the lift-controlled maneuver is advantageous, whereas above this value the drag-only maneuvers are advantageous.

To find the $\Delta\omega$ below which the lift-controlled maneuver is advantageous, we equate Eq. (7) to Eq. (9) and to Eq. (10):

$$\Delta\omega_c = \frac{2n(1+e)}{n(1+e)-1+e}\left[1-\sqrt{\frac{n(1+e)+1-e}{2n(1+e)}}\right]\left(\frac{C_L}{C_D}\right) \quad \text{for } e<e_c \qquad (11a)$$

$$\Delta\omega_c = \frac{2\sqrt{n(1+e)+1-e}}{n(1+e)-1+e}\left[\sqrt{1+e}-\sqrt{2} + (1-e)\sqrt{\frac{1}{n(1+e)+1-e}}\right]\left(\frac{C_L}{C_D}\right) \quad \text{for } e>e_c \qquad (11b)$$

For $n = 1$ (i.e., when the perigee grazes the atmosphere), we get

$$\Delta\omega_c = (C_L/C_D)[1+e-\sqrt{(1+e)}]/e \qquad \text{for } e<e_c$$

$$\Delta\omega_c = (C_L/C_D)\{\sqrt{[2(1+e)]}-1-e\}/e \qquad \text{for } e>e_c \qquad (12)$$

Figure 2 shows, on an e vs $\Delta\omega$ envelope, the regions where each type of aeroassisted maneuver is more fuel economic. The envelopes were calculated for a maximum lift-to-drag ratio of three.

Exact Solution

We now solve, numerically, the complete aeroassisted lift-controlled maneuver.

The first phase of the maneuver is the transfer from the initial apogee to the atmospheric entry point. This portion is a Keplerian orbit. The velocity impulse ΔV_1 is given by Eq. (6). Using the definition $n \triangleq r_p/r_p'$, we get

$$\Delta V_1 = \sqrt{\frac{\mu}{p}}(1-e)\left[1-\sqrt{\frac{2}{n(1+e)+1-e}}\right] \qquad (13)$$

The second phase is the atmospheric trajectory. Following Vinh,[8] the normalized equations of motion, assuming an exponential atmosphere, are given by

$$\frac{dz}{d\ell} = z\tan\gamma \qquad z(0) = z_i \qquad (14a)$$

$$\frac{du}{d\ell} = -\frac{u(1+\lambda^2)}{E^* z\cos\gamma} - 2\tan\gamma \qquad u(0) = u_i \qquad (14b)$$

$$\frac{d\gamma}{d\ell} = -\frac{\lambda}{z\cos\gamma} - \frac{1}{u} + \frac{1}{\beta r} \qquad \gamma(0) = \gamma_i \qquad (14c)$$

where $z \triangleq 2m\beta/(\rho S C_L^*)$, $u \triangleq \beta V^2/g$, and γ are the dimensionless state variables associated with the altitude, velocity, and flight-path angle, respectively. The independent variable ℓ is the dimensionless arc length defined as $\ell \triangleq \int \beta V \cos\gamma \, dt$.

The β is the inverse of the scale height in the atmospheric model

$$\frac{d\rho}{\rho} = -\beta \, dr$$

The exponential atmosphere model is used here, instead of a more accurate model, since the purpose is to achieve ΔV vs $\Delta\omega$ relationships, rather than to solve accurately a single trajectory. The approximate solution [Eq. (7)] is independent of the density, thereby suggesting that ΔV vs $\Delta\omega$ is not sensitive to atmospheric variations.

The initial conditions to Eqs. (14) are derived as follows.

z_i is derived from the entry altitude R_a:

$$z_i = 2m\beta/[\rho(R_a)SC_L^*] \qquad (15)$$

The u_i and γ_i are the nondimensional velocity and flight-path angle at the entry point. Their values depend on the magnitude of the first velocity impulse ΔV_1, which is a control variable, and is given by

$$u_i = \frac{\beta V_i^2}{g} = (2\mu\beta/g)\left(\frac{1}{R_a} - \frac{1}{r_a + r_p'}\right) = \frac{2\mu\beta}{ag}\left[\frac{a}{R_a} - \frac{n}{n(1+e)+1-e}\right]$$

$$\cos\gamma_i = \sqrt{\frac{r_a r_p'}{R_a(r_a + r_p' - R_a)}} = \sqrt{\frac{a(1-e^2)}{R_a[n(1+e)+1-e-nR_a/a]}} \qquad (16)$$

The λ is the dimensionless control variable, defined as $\lambda \triangleq C_L/C_L^*$, where C_L^* is the lift coefficient for which the ratio C_L/C_D is maximum. E^* is defined as $(C_L/C_D)_{\max}$. The C_L is limited by the maximum lift coefficient; hence $|\lambda| \leq \lambda_{\max}$.

The exit condition is $z(\ell_f) = z_f$. The velocity and the flight-path angle at the exit, u_f and γ_f, are unspecified.

At the exit, a tangential velocity impulse is applied to raise the apogee of the orbit to the initial value r_a.

The velocity impulse ΔV_2 is

$$\Delta V_2 = \sqrt{\frac{2\mu(1-R_a/r_a)}{R_a(1-R_a^2\cos^2\gamma_f/r_a^2)}} - V_f = \sqrt{\mu/p}\left(\sqrt{\frac{2\{1-(R_a)/[a(1+e)]\}(1-e^2)}{(R_a/a)\{1-(R_a^2\cos^2\gamma_f)/[a^2(1+e)^2]\}}} - \frac{V_f}{\sqrt{\mu/p}}\right) \qquad (17)$$

Finally, a third tangential impulse is applied at the apogee r_a to bring the perigee to the initial value r_p:

$$\Delta V_3 = \sqrt{\frac{2\mu}{r_a}}\left[\sqrt{\frac{r_p}{r_a + r_p}} - \sqrt{\frac{(R_a \cos^2\gamma_f / r_a)(1 - R_a/r_a)}{1 - R_a^2 \cos^2\gamma_f / r_a^2}}\right]$$

$$= \sqrt{\mu/p}\left[1 - e - \sqrt{\frac{2(1-e)R_a \cos^2\gamma_f(1 + e - R_a/a)}{a[(1+e)^2 - R_a^2\cos^2\gamma_f/a^2]}}\right] \quad (18)$$

The total velocity change is the sum of Eqs. (13), (17), and (18).

The change of the line of apsides angle $\Delta\omega$ is given by

$$\Delta\omega = \cos^{-1}\left\{\frac{1}{e_i}\left[\frac{2r_a r_p'}{(r_a + r_p')R_a} - 1\right]\right\} + \cos^{-1}\left\{\frac{1}{e_f}\left[\frac{R_a V_f^2 \cos^2\gamma_f}{\mu} - 1\right]\right\} - \frac{\ell_f}{\beta R_a}$$

where e_f is the eccentricity of the exit trajectory after the correction ΔV_2.

Using orbital relationships we express $\Delta\omega$ in terms of the parameters of the original ellipse, a/R_a and e, the ratio n, and the exit conditions V_f,γ_f:

$$\Delta\omega = \cos^{-1}\left\{\left(\frac{n(1+e)+1-e}{n(1+e)-1+e}\right)\left[\frac{2(1-e^2)}{n(1+e)+1-e}\left(\frac{a}{R_a}\right) - 1\right]\right\}$$

$$+ \cos^{-1}\left\{\left[1 - \left(2 - \frac{V_f^2 R_a}{\mu}\right)\frac{V_f^2 R_a}{\mu}\cos^2\gamma_f\right]^{-1/2} \times \left(\frac{V_f^2 R_a}{\mu}\cos^2\gamma_f - 1\right)\right\} - \frac{\ell_f}{\beta R_a} \quad (19)$$

We observe that the total velocity change ΔV_{total} and the rotation $\Delta\omega$ depend on two control variables: the first velocity change ΔV_1 and the lift control law $\lambda(\ell)$.

For every ΔV_1, a specified control law $\lambda(\ell)$ gives an orbital rotation $\Delta\omega$ with a certain fuel cost ΔV_{total}. It is desired to choose the control law that yields the minimum ΔV_{total} for a given rotation.

This optimal control problem is solved numerically using the algorithm presented in Ref. 11. To get an insight into the optimal solution, we note that if we do not impose any additional constraint, then this problem is similar to the optimal glide problem of Ref. 8, for which the optimal control is very close to $\lambda = 1$ (i.e., flight at maximum lift to drag ratio). Actually, λ should approach zero exactly at the exit point, to satisfy the transversality condition for the free γ_f case. However, since the major effect of the aerodynamic control is taking place at the lower layers of the atmosphere (i.e., at the middle portion of the atmospheric trajectory), then the total result is not sensitive to deviations of the control (lift) from the optimal one near the entry or exit points.

The optimal solution for a specific example (see next section) verifies this observation.

Hence the maximum lift-to-drag ratio will be used to control the spacecraft during the atmospheric pass. Negative lift ($\lambda = -1$) leads to a forward rotation (i.e., the line of apsides rotates in the same direction as the motion), whereas positive lift ($\lambda = 1$) leads to a backward rotation of the line of apsides.

The equations of motion (14) will be integrated using $\lambda = 1$. For every entry angle γ_i (which is equivalent to a certain ΔV_1), the solution of Eqs. (14–19) yields the "optimal" ΔV_{total} and the corresponding $\Delta\omega$. By varying γ_i, the envelope ΔV_{total} vs $\Delta\omega$ is generated.

If we impose a constraint on the exit angle γ_f, the optimal solution for λ should deviate slightly from $\lambda = 1$ since the control acts to match the specified final conditions. This case is expected to be more fuel consuming than the unconstrained case. This also is discussed, in the next section, for a specific example.

Limitations on the Rotation

An issue of practical importance is the maximum rotation angle $\Delta\omega$ that can be achieved in a single pass. This angle is limited by the maximum permissible entry angle $(\gamma_i)_{max}$. Clearly, if γ_i is too negative, then the spacecraft does not exit the atmosphere. The maximum negative value of γ_i is then the one for which the exit angle γ_f is zero, or, alternatively, the one for which the aerodynamic heating exceeds the maximum allowed.

If an orbital rotation change greater than $(\Delta\omega)_{max}$ is desired, then several atmospheric passes are performed. After each pass, the second correction impulse ΔV_2 is applied. The third impulse, ΔV_3, is applied after the last pass is completed.

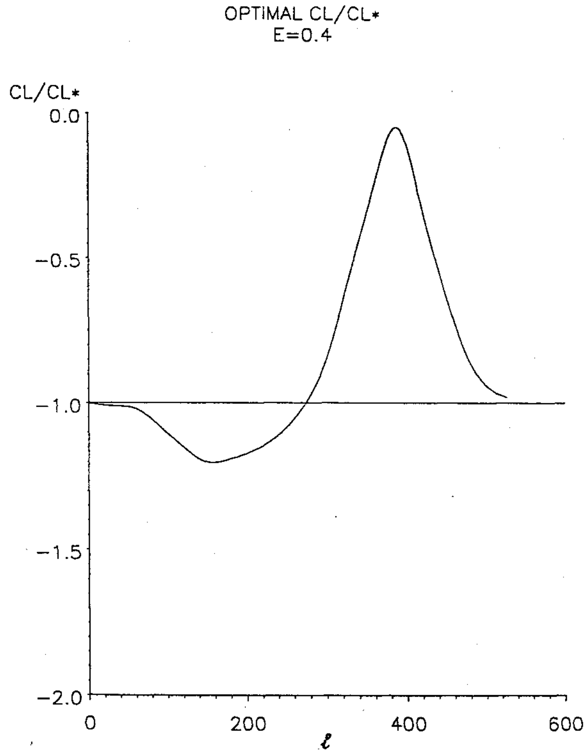

Fig. 3 Optimal lift control law for $\gamma_f = -\gamma_i$, $\Delta\omega = 11.6$ deg.

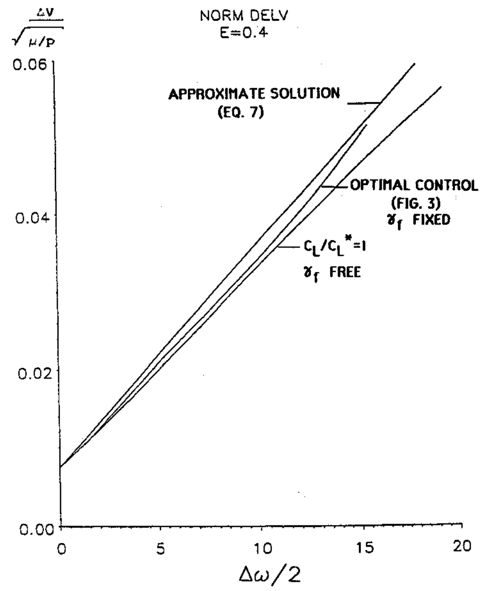

Fig. 4 Total velocity change vs $\Delta\omega$ for fixed γ_f and for free γ_f.

Numerical Examples

The equations were solved for two sets of spacecraft data, as given in Table 1.

The value of E^* (i.e., the ratio $C_L/C_{D_{\max}}$) was assumed to be constant. It should be noted that this ratio actually decreases with altitude, due to viscous effects and drag increase. However, since the major portion of the maneuver takes place in the lower layers of the atmosphere, then it is reasonable to follow the constant E^* assumption.

The atmospheric scale height was taken as

$$\beta R_a = 900$$

Table 1 Spacecraft data

Spacecraft	E^{a}	m/S, kg/m^2	C_L^*
Ref. 3	2.89	333	0.098
Ref. 9	2.36	333	0.151

where R_a, the effective upper limit of the atmosphere, was taken at an altitude of 100 km.

The perigee of the initial orbit was fixed at an altitude of 150 km. The eccentricity was varied from 0.2 to 0.8.

The optimal control numerical solution for the free-exit-angle case was found to be very close to $\lambda = -1$ (for positive $\Delta\omega$). The solution failed to predict the sharp decrease of λ toward zero at the end (to satisfy the transversality condition). This failure is attributed to the insensitivity of the trajectory to lift variations at the higher portion of the trajectory. Because of this insensitivity, the velocity loss ΔV_{total} should be well approximated by the constant $(C_L/C_D)_{\max}$ solution.

The case of a specified exit angle was solved for $\gamma_f = -\gamma_i$. The optimal control $\lambda(\ell)$ is shown in Fig. 3 for an eccentricity of 0.4 and line of apsides rotation of $\Delta\omega = 11.6$ deg. The control deviates from the $\lambda = -1$ value (the optimal control for the γ_f free case). This deviation represents the amount of control necessary to match the exit angle to the specified value.

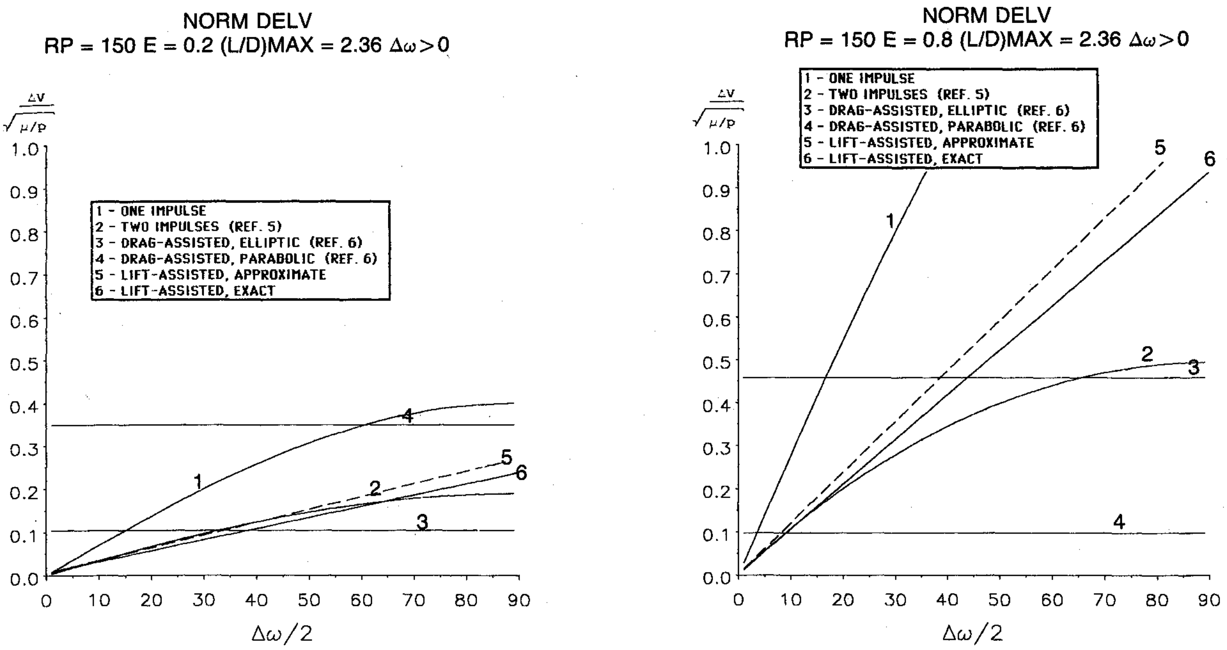

Fig. 5a Comparison of total velocity change vs Δω for $(L/D)_{\max} = 2.36$.

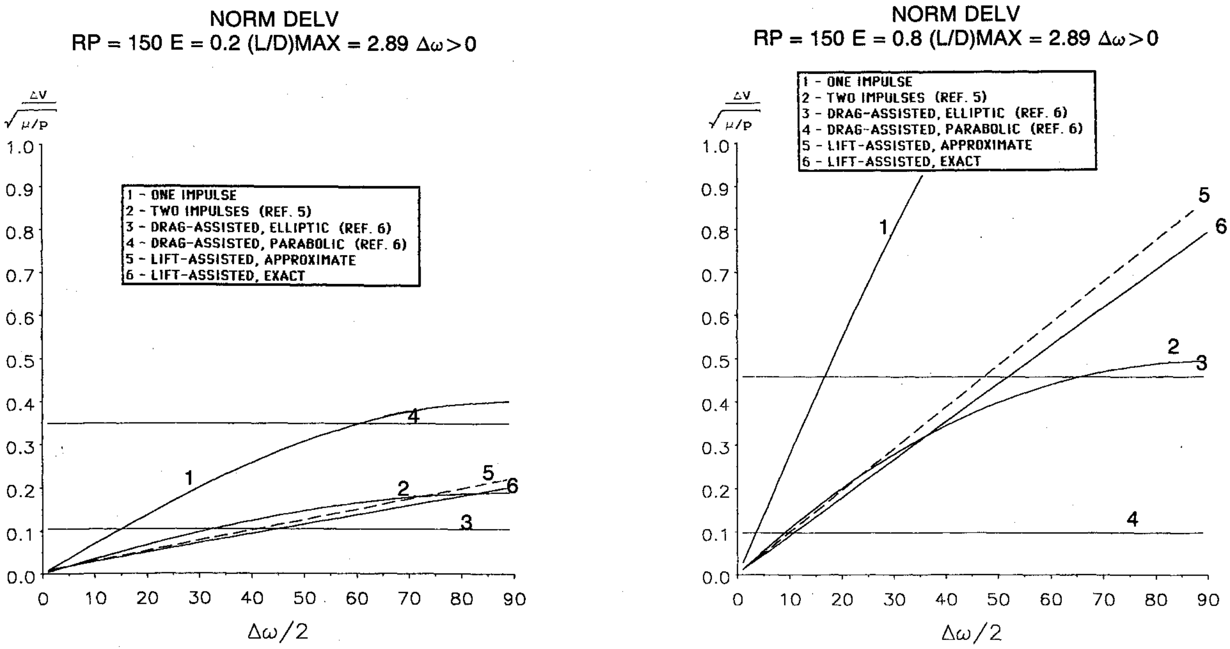

Fig. 5b Comparison of total velocity change vs Δω for $(L/D)_{\max} = 2.89$.

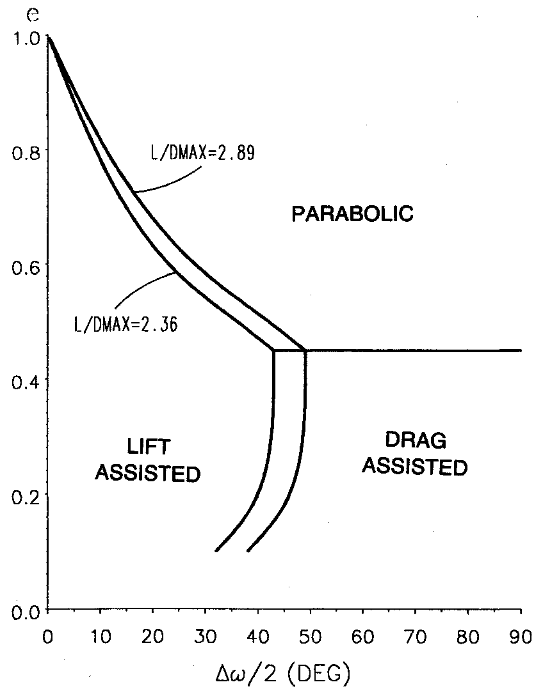

Fig. 6 Optimal regions—exact solution.

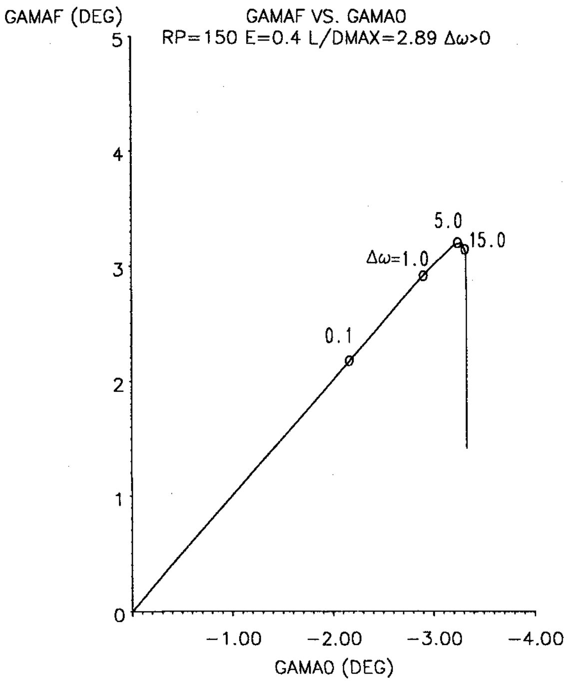

Fig. 7 Exit angle vs entry angle, $(L/D)_{max} = 2.89$.

In Fig. 4, the velocity loss vs $\Delta\omega$ is shown for the γ_f free case ($\lambda = -1$) and γ_f fixed case. The velocity loss for both cases is well approximated by the $\lambda = -1$ solution. The approximate solution discussed previously is also presented in Fig. 4, and is in good agreement with the exact solution.

Since the $(C_L/C_D)_{max}$ solution was found to yield good approximation to the optimal solution for ΔV, the complete mapping of ΔV vs $\Delta\omega$ was done using this control.

The total nondimensional ΔV required for positive changes of the rotation angle is shown in Figs. 5. For comparison, the values of ΔV for other maneuvers (two-impulse propulsive maneuver, drag-only maneuver, parabolic maneuver) are shown. Also, the result of the approximation discussed previously is shown.

First, we observe that the lift-control, aeroassisted maneuver is more fuel-economic than the pure-propulsive maneuver. This was expected, since the maximum L/D ratio is larger than two. Next, we compare each one of the aeroassisted methods. As discussed before, the drag-only, aeroassisted maneuver does not depend on $\Delta\omega$, whereas the lift-assisted maneuver does. Hence, for each value of eccentricity e, there is a certain $\Delta\omega$ below which the lift-assisted maneuver is superior.

The envelope e vs $\Delta\omega$ is shown in Fig. 6. In this figure, the regions where each maneuver is more fuel economic are shown. It is clear that the lift-assisted maneuver is superior at an appreciable range of parameters (up to 110 deg at eccentricities less than 0.4).

Further, we discuss the maximum rotation that can be achieved in a single pass. in Fig. 7 we show the exit angle γ_f vs the atmospheric entry angle γ_i. As γ_i becomes more negative, the atmospheric portion of the trajectory is increased, and the maximum dynamic pressure is increased. Hence a larger rotation angle is achieved. The corresponding velocity loss in the atmosphere is increased, and the exit angle is decreased. At a certain point, as seen in Fig. 7, a sharp increase in the sensitivity of γ_f to γ_i indicates the practical limit to the rotation change in a single pass. For this particular example, it is not practical to enter the atmosphere at angles steeper than 3.33 deg. Reference 10, which discusses entry maneuvers using drag only, shows similar results. The maximum allowed heating constraint, which is characteristic to each application, may impose a further limit on this angle. The corresponding line of apsides angle change is about 15 deg. When larger changes are desired, several atmospheric passes should be performed.

Conclusions

A lifting vehicle can perform a lift-controlled aeroassisted maneuver as well as drag-only maneuver. In this paper, the aeroassisted, lift-controlled, coplanar rotation of the line of apsides was analyzed and compared with other maneuvers. It was shown that for spacecraft with a lift-to-drag ratio greater than two, the lift-control, aeroassisted maneuver is more fuel-economic than the pure-propulsive maneuver. Compared to other aeroassisted maneuvers that use only drag, the lift-assisted maneuver is superior up to a certain value of $\Delta\omega$, which depends on the eccentricity e. Envelopes of e vs $\Delta\omega$ were presented.

When the desired line of apsides rotation angle is less than a certain value (about 15 deg for a typical example), then the orbital change can be performed in a single atmospheric pass. Otherwise, several passes are needed.

Appendix: Approximate Formulas for Pure-Propulsive Transfer

A method for determining the pure-propulsive transfer is discussed by Vinh and Johannesen.[6] It is shown that the optimal transfer between two elliptical orbits with the same shape and size, whose major axes are rotated by an angle of $\Delta\omega$, is given by the solution of the equations

$$e_1^2 = \frac{Y^2(X+Y+1)^2(X-1)}{X^2(X+1)(X+2)} + \frac{(Y^2-X)^2}{X^2} \tag{A1}$$

$$(X-1)[XY^2-(X+1)Y-X(X+1)]\tan\left(\frac{\Delta\omega}{2}\right)$$
$$= (X+1)[(2X+1)Y^2+(X^2-1)Y-X(X+2)] \times \sqrt{\frac{(X-1)}{(X+1)(X+2)}} \tag{A2}$$

where X and Y are given by

$$X = \frac{1}{1+e\cos f_1}, \qquad Y = \sqrt{p_1/p}$$

The e_1 and e are the eccentricities of the initial orbit and the transfer orbit, respectively, p_1 and p are the parameters of the initial orbit and the transfer orbit, respectively, and f_1 is the true anomaly on the transfer orbit where the first impulse is applied.

The total minimum characteristic velocity for the two-impulse transfer is given by Ref. 6:

$$\frac{\Delta V}{\sqrt{\mu/p_1}} = \frac{2Y(1-Y)}{X}\sqrt{1+\frac{X-1}{(X+1)(X+2)}} \tag{A3}$$

Define

$$X = 1 + \Delta X$$

$$Y = 1 - \Delta Y$$

A numerical investigation of Eqs. (A1) and (A2) shows that a considerable range of e_1 and $\Delta\omega$ can be spanned by small variations of ΔX and ΔY. In particular, if we limit e_1 to 0.4 and $\Delta\omega$ to 50 deg, then ΔX and ΔY are <0.1. Following this restriction of e_1 and $\Delta\omega$, we expand Eqs. (A1) and (A2), keeping only first-order terms of ΔX and ΔY.

Hence

$$e_1^2 = \frac{(1-\Delta Y)^2(3+\Delta X-\Delta Y)^2\Delta X}{(1+\Delta X)^2(2+\Delta X)(3+\Delta X)} + \frac{[(1-\Delta Y)^2-(1+\Delta X)]^2}{(1+\Delta X)^2}$$

$$= \frac{3}{2}\Delta X + O(\Delta X^2, \Delta Y^2, \Delta X\Delta Y) \tag{A4}$$

$$\Delta X[(1+\Delta X)(1-\Delta Y)^2 - (2+\Delta X)(1-\Delta Y)$$

$$-(1+\Delta X)(2+\Delta X)]\tan\frac{\Delta\omega}{2}$$

$$=(2+\Delta X)[(3+2\Delta X)(1-\Delta Y)^2+(2\Delta X+\Delta X^2)(1-\Delta Y)$$

$$-(1+\Delta X)(3+\Delta X)]\sqrt{\frac{\Delta X}{(2+\Delta X)(3+\Delta X)}} \tag{A5a}$$

$$[-3\Delta X + O(\Delta X^2, \Delta Y^2, \Delta X\Delta Y)]\tan\frac{\Delta\omega}{2}$$

$$= \sqrt{6\Delta X}\,[-2\Delta Y + O(\Delta X\Delta Y, \Delta Y^2)] \tag{A5b}$$

For small ΔX, ΔY, we get from Eqs. (A4) and (A5):

$$\Delta X = \frac{2}{3}e_1^2 \tag{A6a}$$

$$\Delta Y = \left[\sqrt{6\Delta X}\tan\left(\frac{\Delta\omega}{2}\right)\right]\Big/4 = \frac{e_1}{2}\tan\left(\frac{\Delta\omega}{2}\right) \tag{A6b}$$

The total velocity is given by Eq. (A3):

$$\frac{\Delta V}{\sqrt{\mu/p_1}} = \frac{2\Delta Y(1-\Delta Y)}{1+\Delta X}\sqrt{1+\frac{\Delta X}{(2+\Delta X)(3+\Delta X)}}$$

$$= 2\Delta Y + O(\Delta X\Delta Y, \Delta Y^2)$$

Substituting Eq. (A6) yields

$$\frac{\Delta V}{\sqrt{\mu/p_1}} = e_1\tan\left(\frac{\Delta\omega}{2}\right)$$

For $\Delta\omega < 50$ deg, we get the approximate relation

$$\frac{\Delta V}{\sqrt{\mu/p_1}} = \frac{e_1\Delta\omega}{2}$$

References

[1]London, H. S., "Change of Satellite Orbit Plane by Aerodynamic Maneuvering," *Journal of Aerospace Sciences*, Vol. 29, March 1962, pp. 232–332.

[2]Walberg, G. D., "A Survey of Aeroassisted Orbit Transfer," *Journal of Spacecraft and Rockets*, Vol. 22, No. 1, 1985, pp. 3–18.

[3]Mease, K. D., and Vinh, N. X., "Minimum-Fuel Aeroassisted Coplanar Orbit Transfer Using Lift Modulation," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 1, 1985, pp. 134–141.

[4]Speyer, J. L., and Womble, M. E., "Approximate Optimal Atmospheric Entry Trajectories," *Journal of Spacecraft and Rockets*, Vol. 8, No. 11, 1971, pp. 1120–1125.

[5]Marec, J. P., *Optimal Space Trajectories*, Elsevier, Amsterdam, 1979.

[6]Vinh, N. X., and Johannesen, J. R., "Optimal Aeroassisted Transfer Between Coplanar Elliptical Orbits," *Acta Astronautica*, Vol. 13, Nos. 6 and 7, 1986, pp. 291–299.

[7]Bate, R. R., Mueller, D. D., and White, J. E., *Fundamentals of Astrodynamics*, Dover, New York, 1971.

[8]Vinh, N. X., *Optimal Trajectories in Atmospheric Flights*, Elsevier, New York, 1981.

[9]Hull, D. G., Giltner, J. M., Speyer, J. L., and Mapar, J., "Minimum Energy-Loss Guidance for Aeroassisted Orbital Plane Change," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 4, 1985, pp. 487–493.

[10]Vinh, N. X., Johannesen, J. R., Longuski, J. M., and Hanson, J. M., "Second-Order Analytic Solution for Aerocapture and Ballistic Fly-Through Trajectories," *Journal of the Astronautical Sciences*, Vol. 32, No. 4, 1984, pp. 429–445.

[11]Bryson, A. E., and Ho, Y. C., *Applied Optimal Control*, Hemisphere, Washington, DC, 1975, pp. 71–75.